Session 2:

The Algorithmic Trading Process

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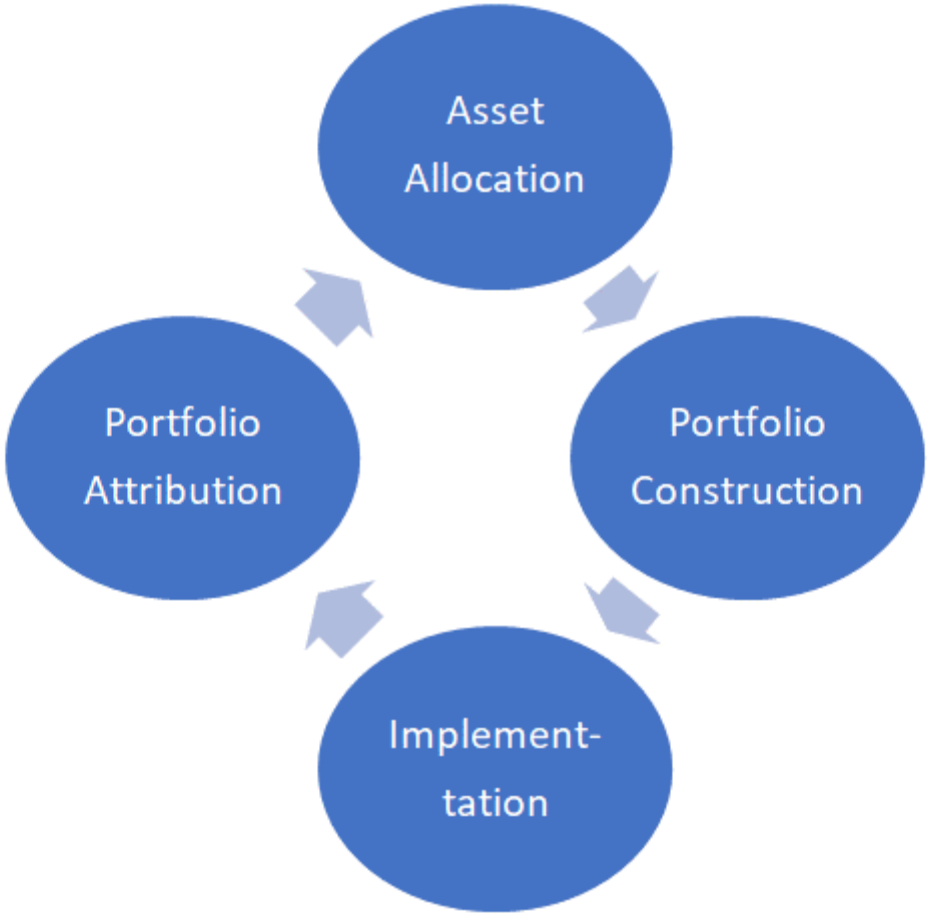
Session 2: Outline

1. Algorithmic Trading Process
    - Strategy Selection (Macro & Micro)
    - Order Submission Rules (LOM & SOR)
2. Trading Costs
    - Formulas
    - Data Requirements
3. Market Impact Models
    - Market Observations
    - Estimating MI Parameters
    - Model Evaluation
4. Traders Dilemma
    - Minimizing Trade Cost
    - Efficient Trade Strategies
    - Optimal POV Rates
5. Python Examples
    - TCA Scripts
6. Examples
    - Trade Costs
7. Trading Analytics
    - Pre-Trade
    - Cost Curves
    - Post-Trade

Algorithmic Trading Process

Investment Cycle



Implementation

- Investment Goals
- Macro Decisions
- Micro Decisions
- Limit Order Models
- Smart Order Routing

Investment Goals

- This phase determines the investment portfolio and ultimately the “trade list” or “basket” based on portfolio rebalances, cash inflows, redemptions, or combinations of all three.

Macro Decisions

- Macro Decisions = how the order is to be sliced into smaller pieces and traded over time.
- This is also known as the trading strategy, trading schedule, trade trajectory, etc.
- Investors manage the tradeoff between cost and timing risk.
- For a trade list (e.g., program), investors manage total portfolio risk (e.g., covariance & volatility).
- Strategies are determined via an optimization process. These include:
  - Traders Dilemma
  - Cost Minimization
  - Price Improvement

Micro Decisions

- Micro Decisions = determine when it is appropriate for an algorithm to deviate from an optimally prescribed strategy and adapt to changing market conditions (e.g., prices, volatility, momentum, etc.)
- Needs to be consistent with the Macro Trading Goal.
- Micro-level decisions require real-time market data, volatility, volumes, and investors execution prices.
- Some Micro Decision criteria include:
  - Target Price
  - Passive-in-the-Money
  - Aggressive-in-the-Money

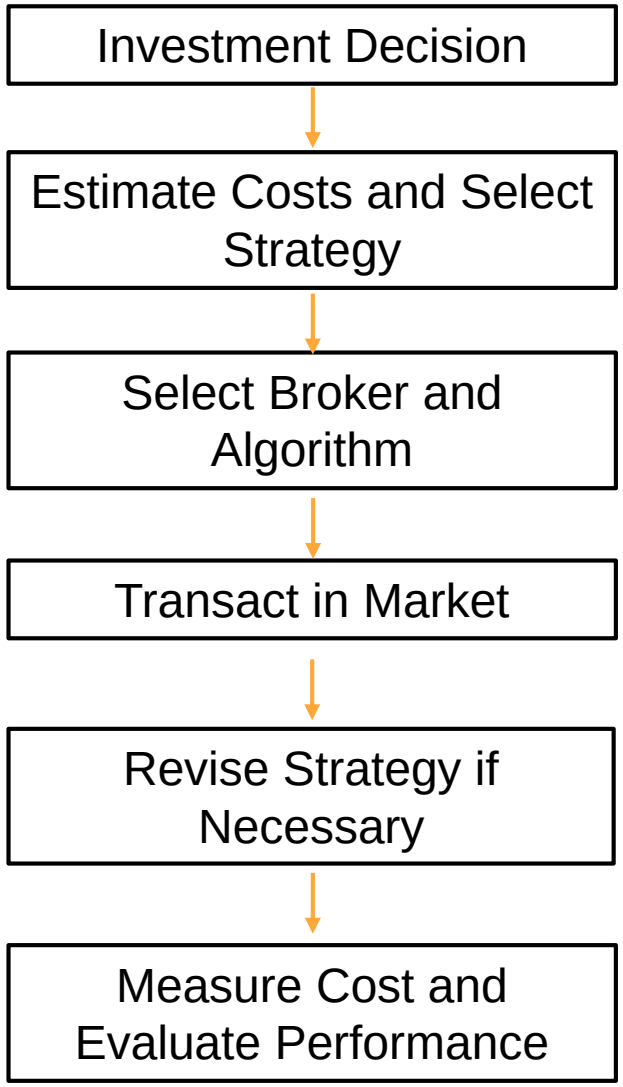
Limit Order Models (LOM)

- Limit Order Model = determine the best combination of limit and market orders to send to the market in order to adhere to the higher-level macro goal.
- For example, if the optimal trading rate is 10% and we forecast 10,000 shares to trade in the next 1 min, we know we need to execute 1,000 shares.
- The limit order model may determine that 300 shares to trade at the market, 300 at the bid+1, 300 at the current bid, and 100 at the bid -1.
- The limit order model estimates are based on real-time market conditions and expectations going forward.

Smart Order Router (SOR)

- Smart Order Router = how and where child orders should be entered into the market. e.g., share quantities, limit or market, wait time between orders, selection of “displayed” vs. “dark” markets.
- The SOR continuously calculate in real-time the likelihood of transacting at the specified prices (determined from the limit order model) at the different venues based on shares, order book queue, and time (speed) of executing.
- The SOR determines the venues to submit the orders.
- The SOR needs to determine when and if it is appropriate to accelerate trading, as well as the best venues submit these orders. The best prices are pre-determined by the Limit Order Model.

Trading Process - Implementation



Trading Costs

Implementation Shortfall (IS)

Implementation Shortfall Components:

- **Delay Cost** = price movement in the stock from the time of the investment decision until the order is released to the market.
- **Execution Cost** = price movement in the stock over the time the order is being executed in the market.
- **Opportunity Cost** = the missed profit opportunity of not being able to execute all shares in the order.
- **Fixed Cost** = all fixed fees charged during implementation of the trade.

Implementation Shortfall (IS)

Formula:

$$IS_{\$} = \underbrace{S \cdot (P_0 - P_d)}_{Delay\ Cost} + \underbrace{X \cdot (P_{avg} - P_0)}_{Execution\ Cost} + \underbrace{R \cdot (P_n - P_0)}_{Opportunity\ Cost} + Fixed\ Cost$$

Variables:

S = Order Shares

X = Executed Shares

R = Unexecuted Shares

P_0 = Arrival Price - mid-point of bid-ask spread at time of order entry

P_d = Decision Price - mid-point of bid-ask spread at time of decision

P_n = End Price - mid-point of bid-ask spread at end time of (completion/cancel)

P_{Avg} = Average Execution Price of the Order

Fixed Cost = Commissions, Taxes, and Fees & Rebates

Spread Costs are embedded in the actual execution price of the stock

Fees & Rebates are often included in the broker commission

Implementation Shortfall (IS)

Historical (Past):

$$IS = \underbrace{\langle Delay\ Costs \rangle}_{\text{Before}} + \underbrace{\langle Trade\ Costs \rangle}_{\text{During}} + \underbrace{\langle Opportunity\ Cost \rangle}_{\text{After}} + \underbrace{\langle Fixed\ Costs \rangle}_{\text{Visible}}$$

Implementation Shortfall (IS)

Historical (Past):

$$IS = \underbrace{\left\langle \begin{matrix} Delay \\ Costs \end{matrix} \right\rangle}_{\text{Before}} + \underbrace{\left\langle \begin{matrix} Trade \\ Costs \end{matrix} \right\rangle}_{\text{During}} + \underbrace{\left\langle \begin{matrix} Opportunity \\ Cost \end{matrix} \right\rangle}_{\text{After}} + \underbrace{\left\langle \begin{matrix} Fixed \\ Costs \end{matrix} \right\rangle}_{\text{Visible}}$$

Forecast (Future):

$$\left\langle \begin{matrix} Trade \\ Costs \end{matrix} \right\rangle = \underbrace{Expected \left\langle \begin{matrix} Spread\ Costs \\ Price\ Appreciation \\ Market\ Impact \\ Timing\ Risk \end{matrix} \right\rangle}_{\text{Estimate/Forecast}}$$

Trade Cost / Transaction Cost Analysis (TCA)

Past

- Implementation Shortfall
- Cost Measure
  - Single Value
- Calculation:
  - $Trade\ Cost = \left(\frac{P_{avg} - P_0}{P_0}\right) \cdot 10^4 bp$

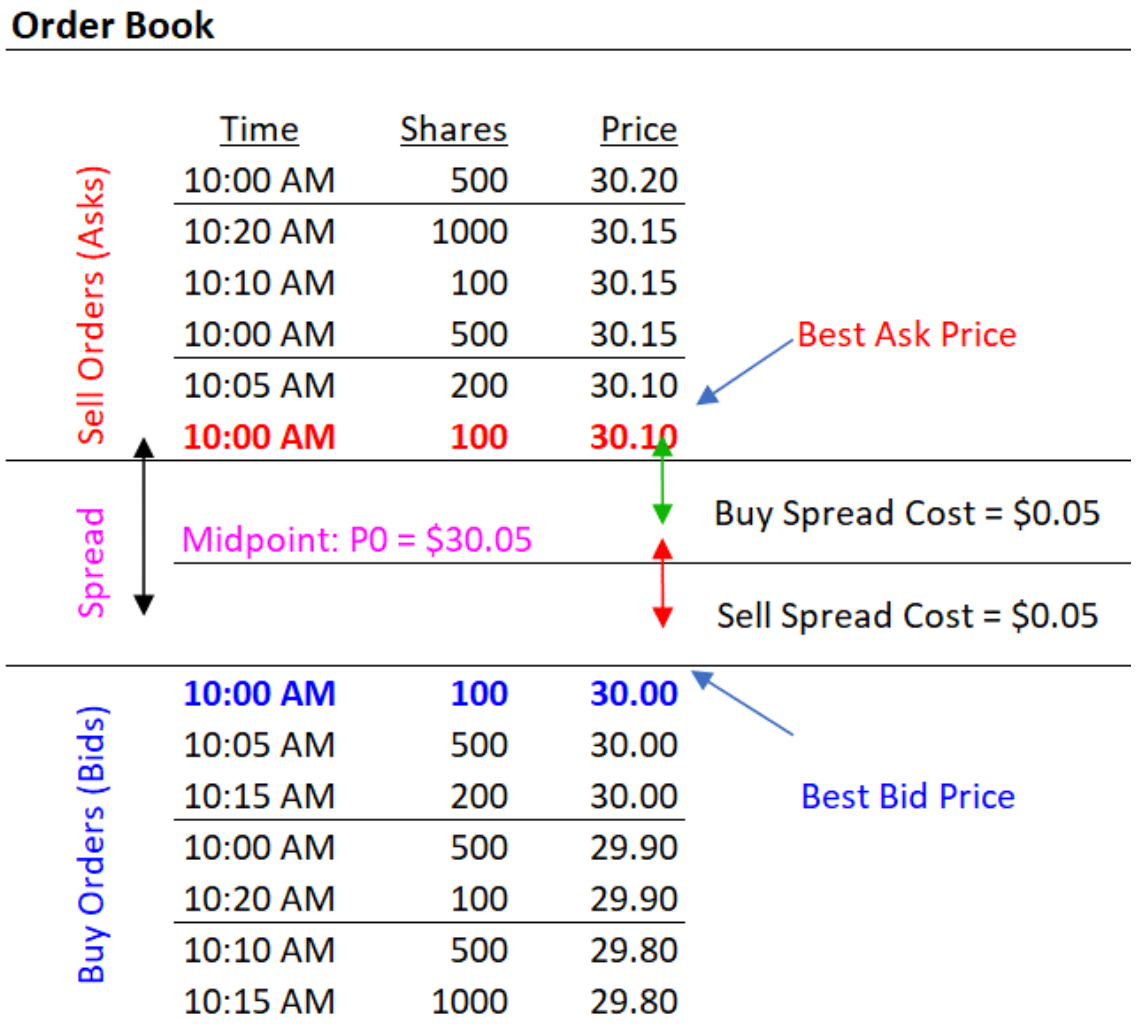
Forward

- Forecast/Estimation
  - Market Impact: MI
  - Price Appreciation: PA
  - Timing Risk: TR
- Distribution of Costs
  - PDF and CDF Graphs
  - Mean = MI + PA
  - Std = TR
- Optimization
  - Optimal Trading Rates

Trading Cost Components

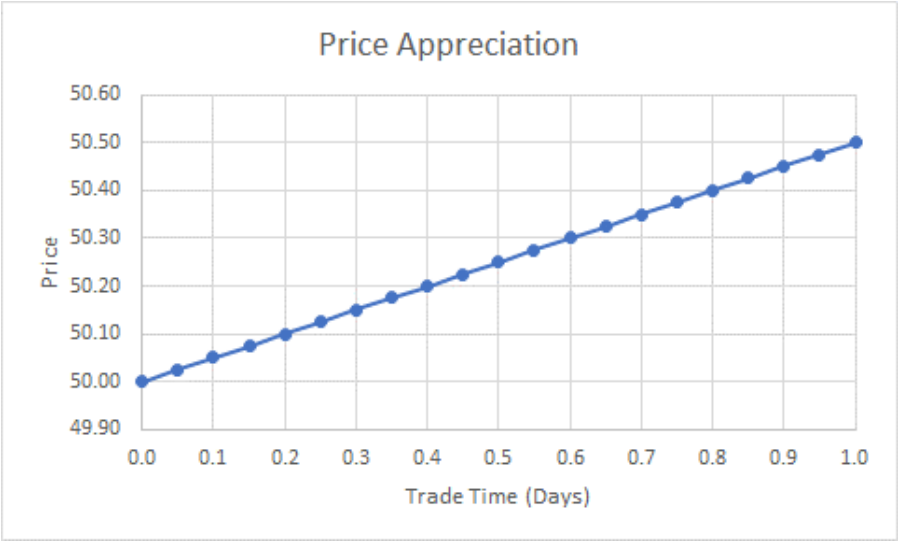
Spread Cost

- **Spread** = difference between best offer (ask) and best bid price.
  - Represent the round-trip cost of transacting small orders, e.g., buying and immediately selling 100 shares.
  - Does not accurately represent the cost of transacting larger share quantities and blocks.
- **Spread Cost** = one-half of the Bid-Ask Spread. Measured as the midpoint of the bid-ask spread to the market price. The spread cost for buy and sell orders are:
  - Buy Order = Ask Price – Mid Point of Bid Ask Spread
  - Sell Order = Mid Point of Bid Ask Spread – Bid Price
- Spread Cost is often incorporated into the Market Impact Cost estimate, i.e., arrival price P0 is taken as the mid-point of the bid-ask spread.



Price Appreciation (PA)

- Price Appreciation (PA) is the natural price movement of the stock. It represents how stock price would evolve in a market without any uncertainty.
- Price appreciation is also referred to as price trend, momentum, drift, or alpha.
- Buy Order = It represents a cost when buying stock in a rising market and a savings when buying stock in a falling market.
- Sell Order = It represents a cost when selling stock in a falling market and a savings when selling stock in a rising market.
- AlphaBp = the estimated price appreciation over one-day expressed in basis points.
- Side = +1 for Buy order, -1 for a Sell order.
- Size = Shares divided by Stock Average Daily Volume (ADV)



Time:

$$PA = Side \cdot \frac{1}{2} \cdot AlphaBp \cdot Time$$

POV Rate:

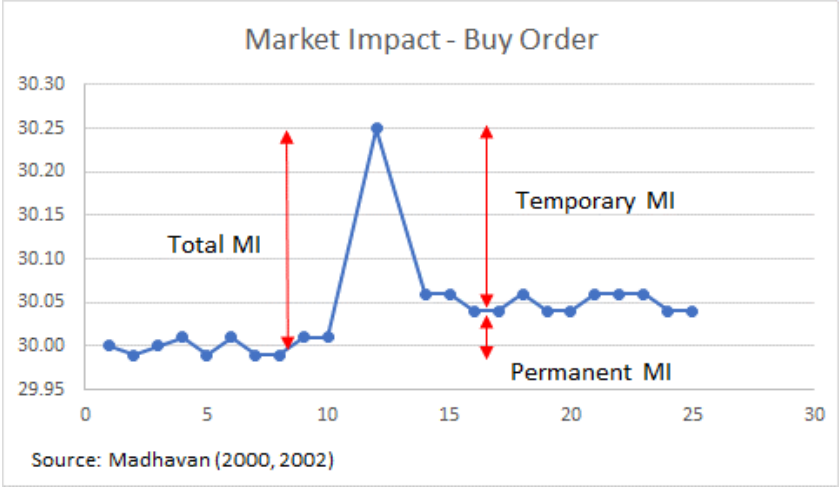
$$PA = Side \cdot \frac{1}{2} \cdot AlphaBp \cdot Size \cdot \frac{(1 - POV)}{POV}$$

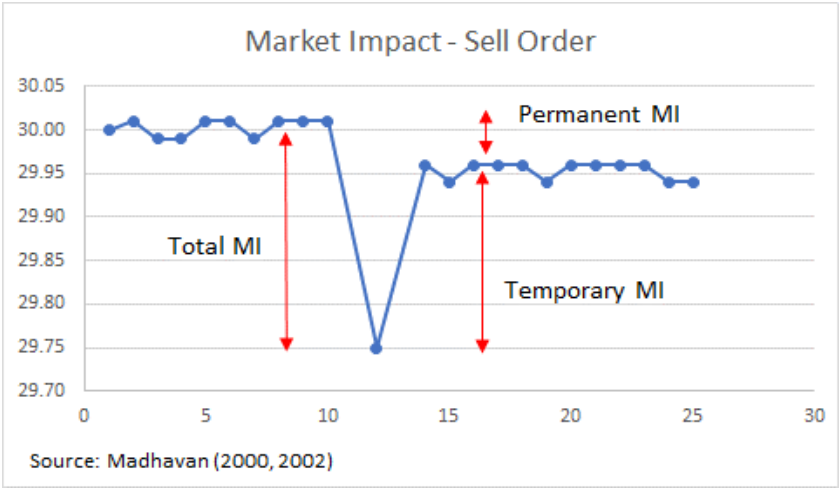
Time-POV Relationship:

$$Trade\ Time = Size \cdot \frac{(1 - POV)}{POV}$$

Market Impact (MI)

- Market Impact (MI)is the price movement in a stock caused by the order or trade. It consists of two components:
  - Temporary Impact = due to the liquidity needs of the investor.
  - Permanent Impact = due to the information content of the trade.
- Mathematically, MI is defined as the difference between the path of the stock with and without the order.
- Heisenberg Uncertainty Principal of Trading.
- Linear Regression Model: Y = b0 + b1*x1 + b2*x2





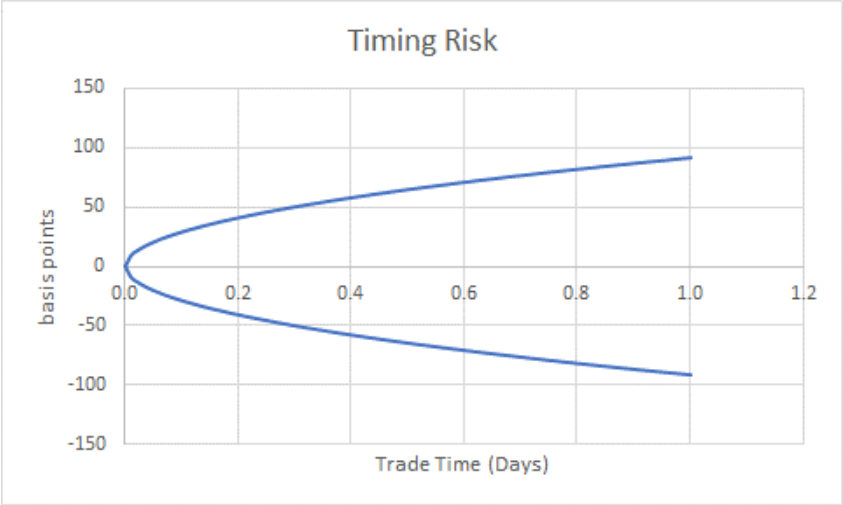
Market Impact (MI)

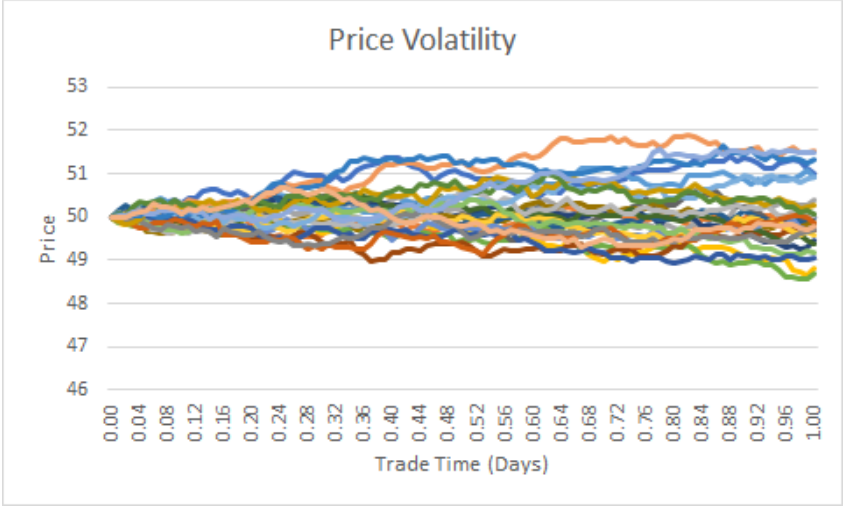
$$I_{bp}^{*} = a_1 \cdot Size^{a2} \cdot \sigma^{a3}$$

$$MI_{bp} = I_{bp}^{*} \cdot \left(b_1 \cdot POV^{a4} + (1 - b_1)\right)$$

Timing Risk (TR)

- Timing Risk (TR) refers to the uncertainty surrounding the estimated trading cost. Timing risk can result in the execution price being higher or lower than the arrival price.
- It consists of three components
  - Price Volatility
  - Volume Uncertainty (Liquidity Risk)
  - Parameter Estimation Error





Timing Risk (TR)

$$TR_{bp} = \sigma \cdot \sqrt{\frac{1}{250} \cdot \frac{1}{3} \cdot \frac{Shares}{ADV} \cdot \frac{1 - POV}{POV}} \cdot 10_{bp}^{4}$$

Trade Cost Equations

Trade Cost Equations

Market Impact (MI):

$$I_{bp}^{*} = a_1 \cdot Size^{a2} \cdot \sigma^{a3}$$

$$MI_{bp} = \left(b_1 \cdot I_{bp}^{*} \cdot POV^{a4} + (1 - b_1) \cdot I_{bp}^{*}\right)$$

My Effect on Stock Price

Timing Risk (TR):

$$TR_{bp} = \sigma \cdot \sqrt{\frac{1}{250} \cdot \frac{1}{3} \cdot Size \cdot \frac{1 - POV}{POV}} \cdot 10_{bp}^{4}$$

Stock Price Movement from everyone else and noise and randomness

Price Appreciation (PA):

$$PA_{bp} = Side \cdot \frac{1}{2} \cdot AlphaBp \cdot Size \cdot \frac{1 - POV}{POV}$$

Stock Price Movement by Itself

Trade Cost Equations

Market Impact (MI):

$$I_{bp}^{*} = a_1 \cdot Size^{a2} \cdot \sigma^{a3}$$

$$MI_{bp} = \left(b_1 \cdot I_{bp}^{*} \cdot POV^{a4} + (1 - b_1) \cdot I_{bp}^{*}\right)$$

Timing Risk (TR):

$$TR_{bp} = \sigma \cdot \sqrt{\frac{1}{250} \cdot \frac{1}{3} \cdot Size \cdot \frac{1 - POV}{POV}} \cdot 10_{bp}^{4}$$

Price Appreciation (PA):

$$PA_{bp} = Side \cdot \frac{1}{2} \cdot AlphaBp \cdot Size \cdot \frac{1 - POV}{POV}$$

Model Parameters:

- a_1, a_2, a_3, a_4, b_1

Variables:

- Size
- Volatility
- Side
- AlphaBp

Decision Variables:

- POV Rate
- Trade Time

Trade Cost Equations

Forecasting Equations - Data

	MI	TR	PA
Parameters	yes	no	no
Size	yes	yes	yes
Volatility	yes	yes	no
AlphaBp	no	no	yes
Side	no	no	yes
POV	yes	yes	yes
Trade Time	yes	yes	yes

Input Variables & Calculations

Trade Quantity:

$$Size = \frac{Shares}{ADV} \quad (\%Adv, decimal)$$

Percentage of Volume:

$$POV = \frac{Shares}{Actual\ Volume\ over\ Trade\ Period} \quad (decimal)$$

(Actual/Historical)

$$POV = \frac{Shares}{Shares + \hat{V}(t)} = \frac{Size}{Size + Time} \quad (decimal)$$

(Forecasted/Predicted)

Trade Time:

$$Time = Size \cdot \frac{(1 - POV)}{POV}$$

Definitions:

$Size = Order\ Size\ expressed\ as\ pct\ of\ ADV$

$ADV = Average\ Daily\ Volume\ (Moving\ Average)$

$POV = Percentage\ of\ Market\ Volume$

$Trade\ Time = Volume\ Time$

$\hat{V}(t) = Predicted\ Volume\ in\ Time\ Period$

$\sigma = Annualized\ Volatility\ (Moving\ Avg, decimal)$

$Side = Order\ Side, +1 = Buy, -1 = Sell$

Input Variables & Calculations

Average Daily Volume:

$$ADV(t) = \frac{1}{n} \cdot \sum_{i=1}^{n} Volume(t-i)$$

Median Daily Volume:

$$MDV(t) = Median\big(Volume(t-1), \dots, Volume(t-n)\big)$$

Annualized Volatility

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

$$\sigma(t) = \sqrt{250 \cdot \frac{1}{n-1} \cdot \sum_{t=1}^{n} (r_t - \bar{r})^2}$$

Forecasted Period Volume:

$$\hat{V}(t) = ADV \cdot f\big(v(t)\big) \cdot Adj$$

Definitions

$ADV = Average\ Daily\ Volume\ (Moving\ Average)$

$MDV = MedianDaily\ Volume\ (Moving\ Average)$

$Volume(t-i) =$ *Actual Total Volume on day t-i*

$\hat{V}(t) = Predicted\ Volume\ in\ Time\ Period$

$P_t = Price\ on\ day\ t$

$r_t = Log\ Return\ on\ day\ t$

$\sigma = Annualized\ Volatility\ (Moving\ Average\ as\ decimal)$

$n = historical\ period, commonly\ 30\ days$

$f\big(v(t)\big)$= *percentage of historical daily volume in period t*

Adj = *volume adjustment due to Day of Week, Month or Quarter End, Earnings, Fed Day, Special Event.*

Measuring Model Performance

$$MSE = \frac{1}{m}\sum_{i=1}^{m}(y_i - \hat{y}_i)^2$$

$$RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(y_i - \hat{y}_i)^2}$$

$$MAD = \frac{1}{m}\sum_{i=1}^{m}|y_i - \hat{y}_i|$$

Shares to Trade from POV Rate

POV Formula:

$$POV = \frac{Shares}{Shares + \hat{V}(t)} = \frac{Shares}{Shares + ADV \cdot f\big(v(t)\big) \cdot Adj}$$

Solve for Shares (in above Formula) :

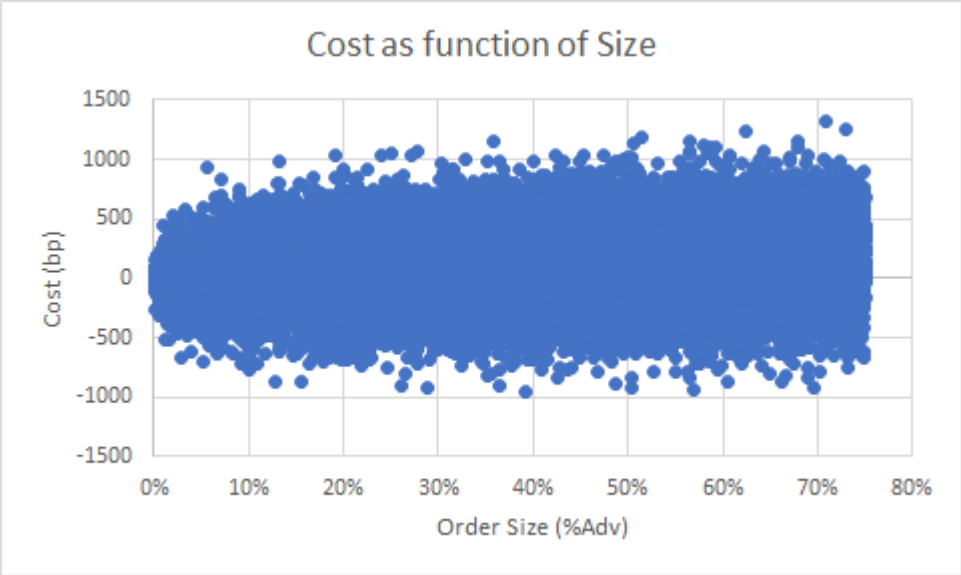
$$Shares = \frac{POV}{1 - POV} \cdot \hat{V}(t) = \frac{POV}{1 - POV} \cdot ADV \cdot f\big(v(t)\big) \cdot Adj$$

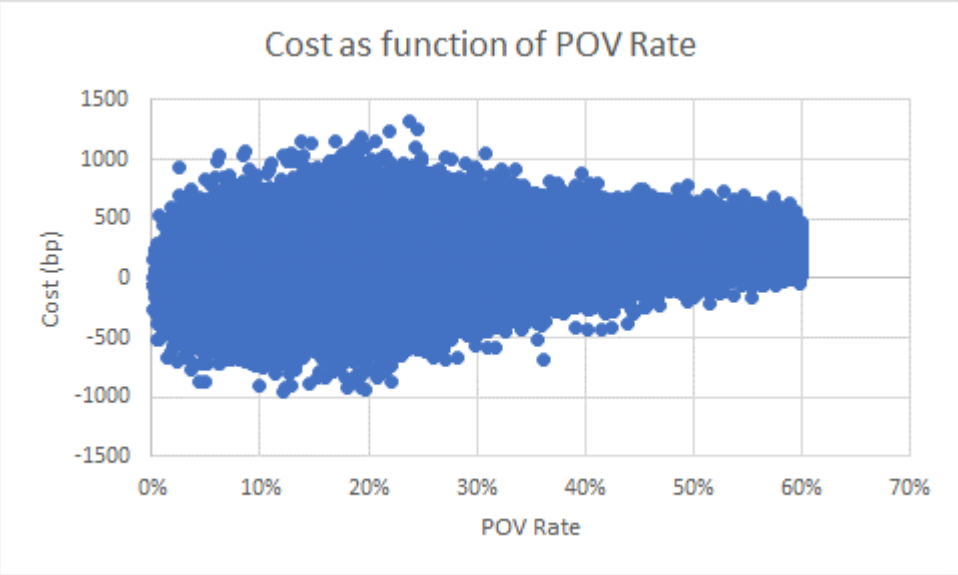
Market Observations

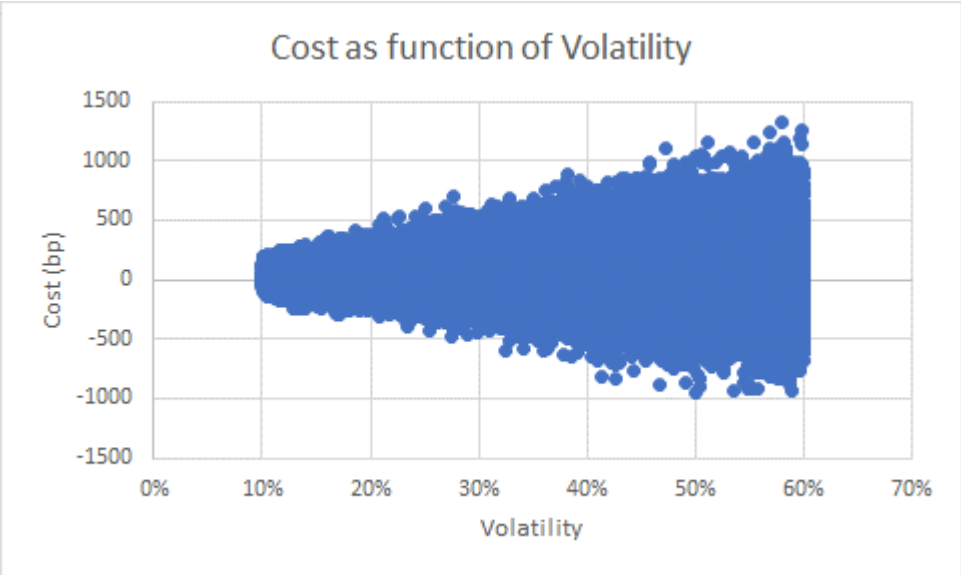
What MI Explanatory Factors are Important?

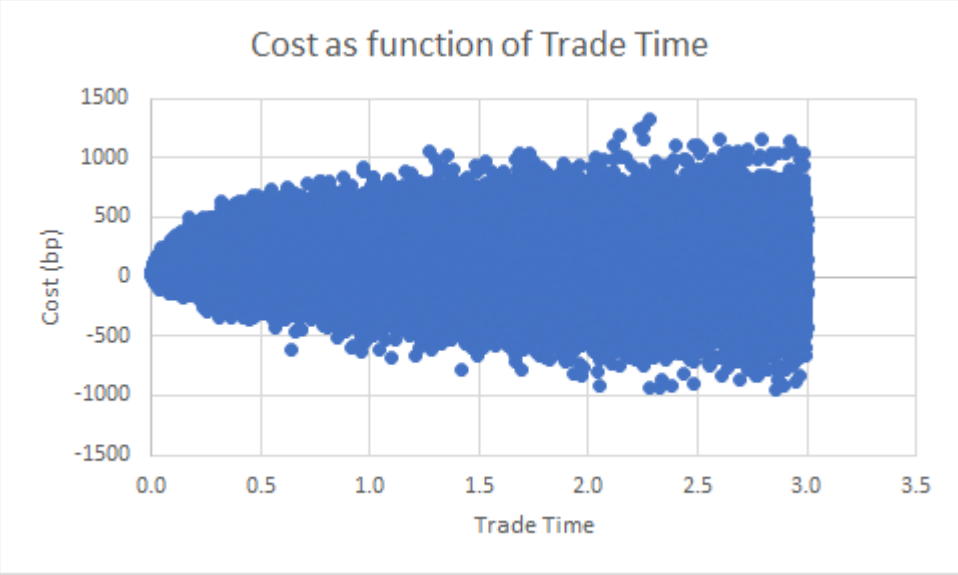
- Share Quantity
  - Size (%Adv)
- Volatility
  - Annualized
- POV
- Trading Rate

Historical Trade Cost Data

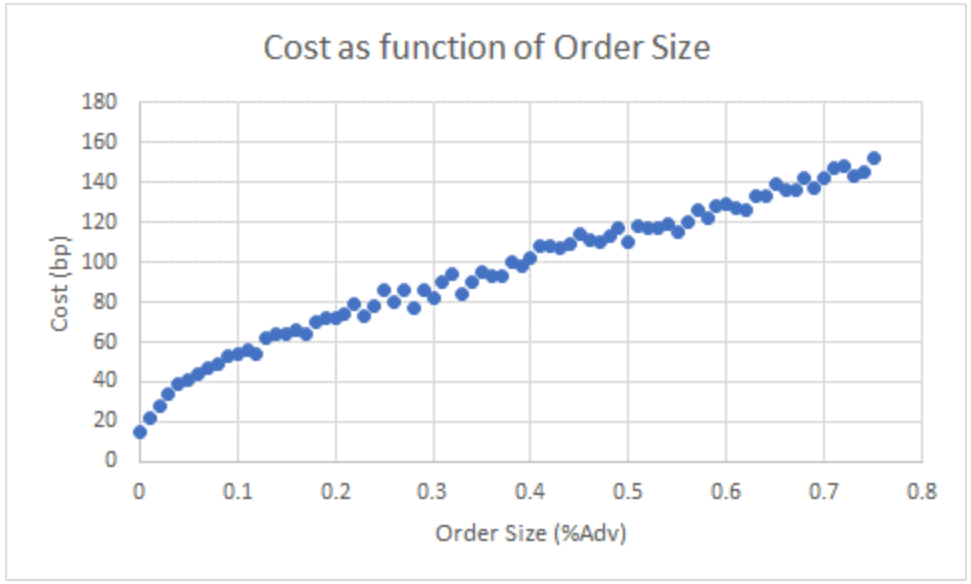


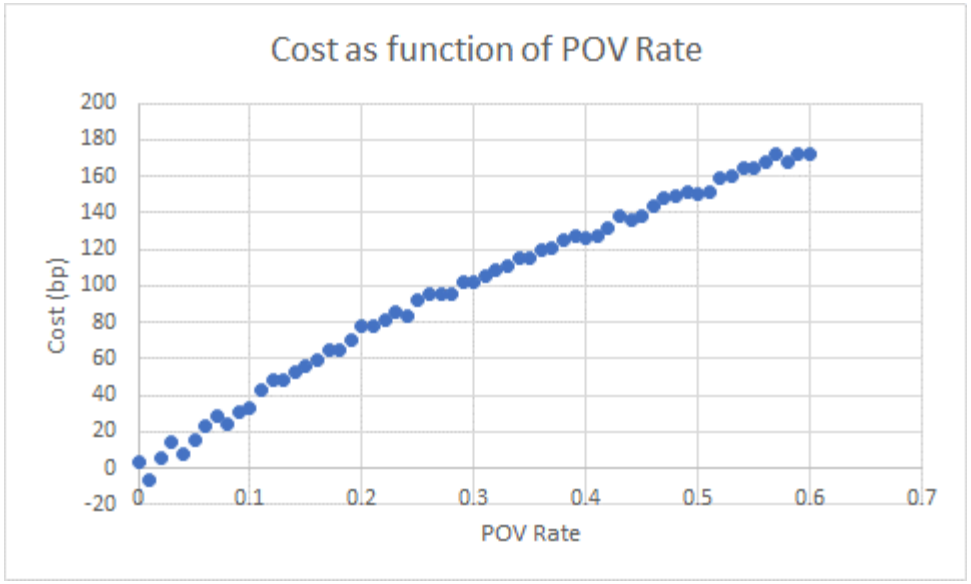


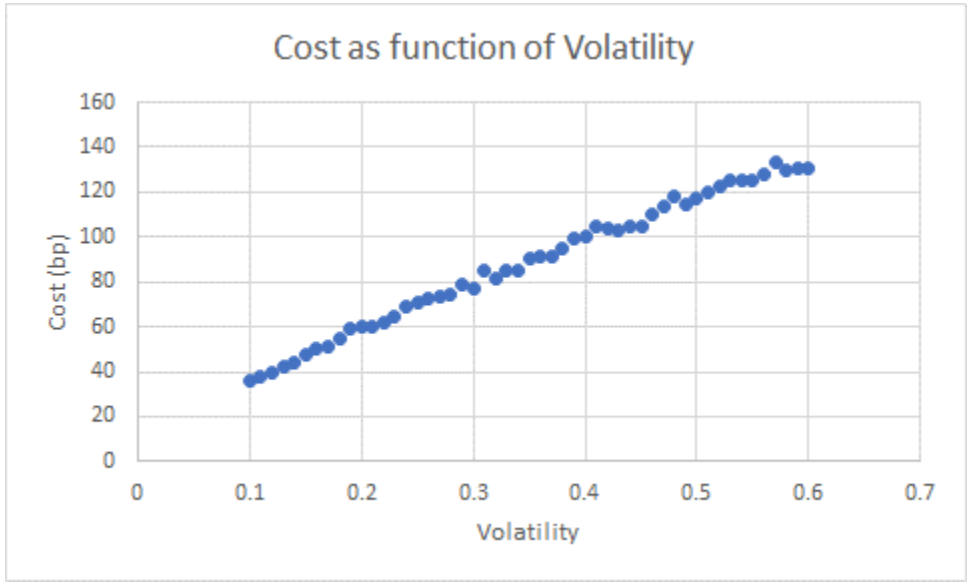


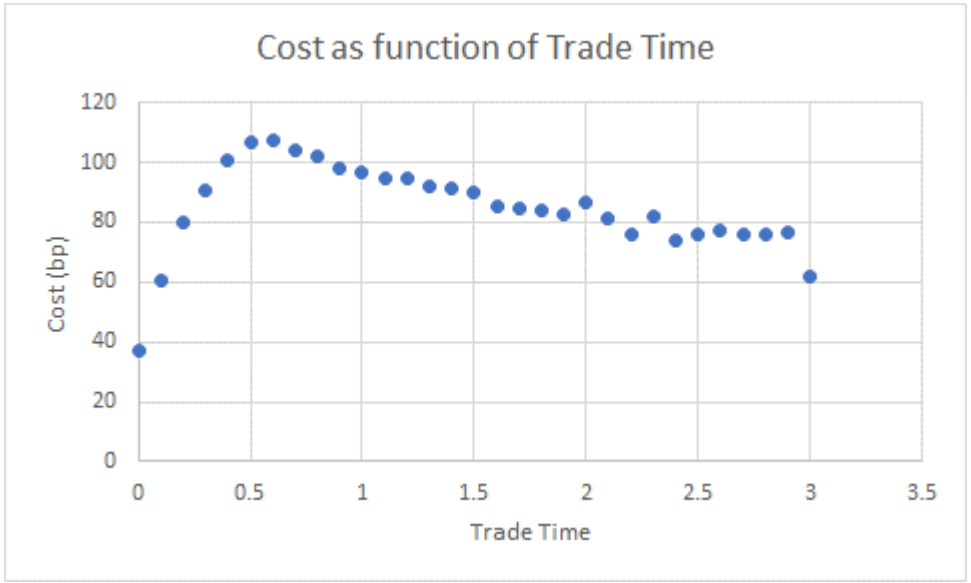


Grouped Trade Cost Data









Developing Market Impact Model

I-Star Model

$$I_{bp}^{*} = a_1 \cdot Size^{a2} \cdot \sigma^{a3}$$

I-Star Model

$$I^*_{bp} = a_1 \cdot Size^{a2} \cdot \sigma^{a3}$$

$$MI_{bp} = \underbrace{b_1 \cdot I^*_{bp} \cdot POV^{a4}}_{\text{Temporary Impact}} + \underbrace{(1 - b_1) \cdot I^*_{bp}}_{\text{Permanent Market}}$$

I-Star Market Impact Model

$$I_{bp}^{*} = a_1 \cdot Size^{a2} \cdot \sigma^{a3}$$

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I-Star Market Impact Model

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I-Star Market Impact Model

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$$MI_{bp} = a_1 \cdot Size^{a2} \cdot \sigma^{a3} \cdot \left(b_1 \cdot POV^{a4} + (1 - b_1)\right)$$

Estimating MI Parameters: Non-Linear Regression

I-Star Model:

$$MI = I^* \cdot \left(b_1 \cdot POV^{a4} + (1 - b_1)\right)$$

$$MI = a_1 \cdot Size^{a2} \cdot \sigma^{a3} \cdot \left(b_1 \cdot POV^{a4} + (1 - b_1)\right)$$

Non-Linear Regression:

$$Min \quad f = \sum (Cost - MI)^2$$

$$Min \quad f = \sum \left(Cost - \left(a_1 \cdot Size^{a2} \cdot \sigma^{a3} \cdot \left(b_1 \cdot POV^{a4} + (1 - b_1)\right)\right)\right)^2$$

Subject to:

$$0 \leq b_1 \leq 1$$

$$a_1, a_2, a_3, a_4 \geq 0$$

where,

Cost = Measured Trade Cost (difference between execution price and arrival price)

Example: Python #1

Estimate Market Impact Parameters using Python

- Python Script: KRG_Est MI_Parameters.ipynb
- File: MIData_KRG.csv

Estimating MI Parameters: Results

MI Parameters	
Parameter	Value
a1	883.25
a2	0.35
a3	0.76
a4	0.83
b1	0.96

I-Star Model:

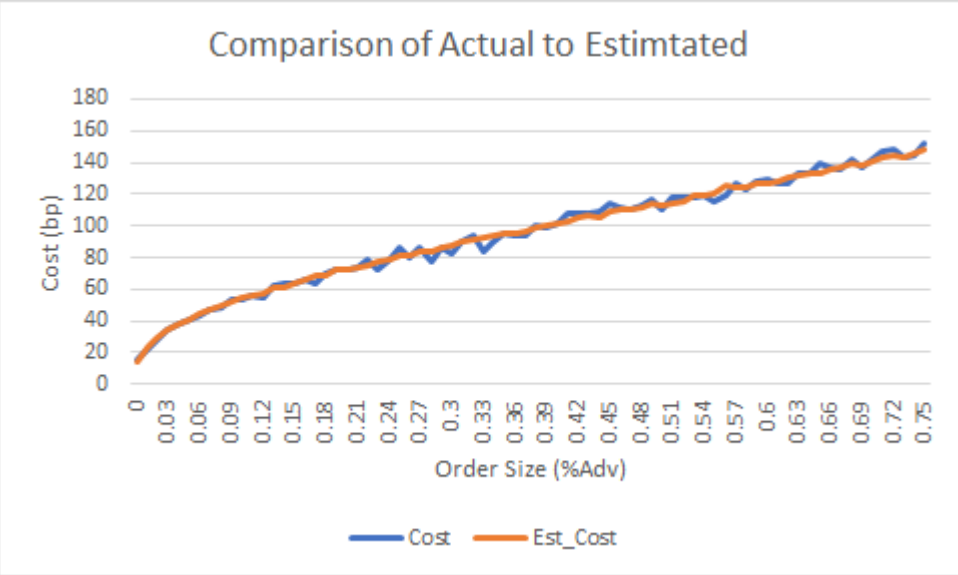
$$I^* = a_1 \cdot Size^{a2} \cdot \sigma^{a3}$$

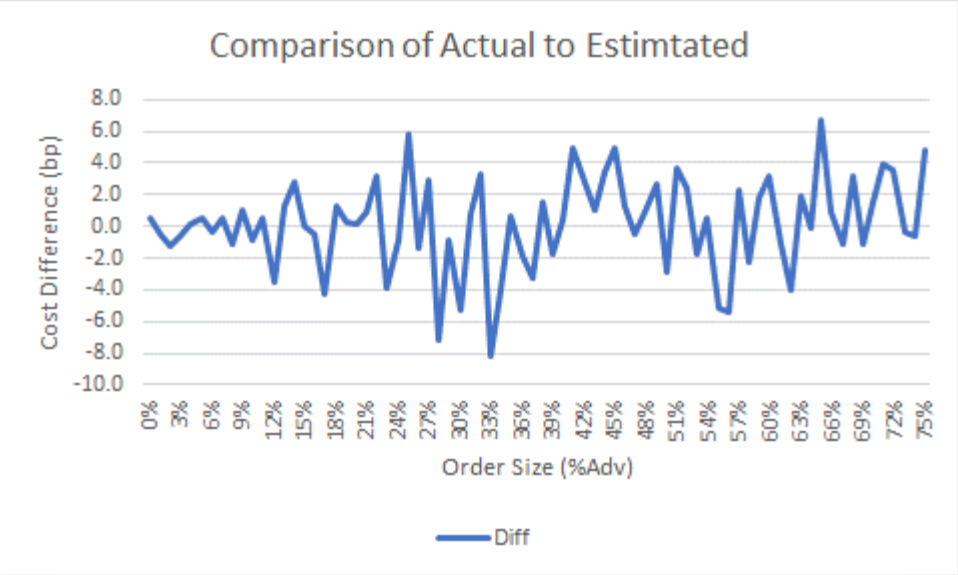
$$MI = I^* \cdot \left(b_1 \cdot POV^{a4} + (1 - b_1)\right)$$

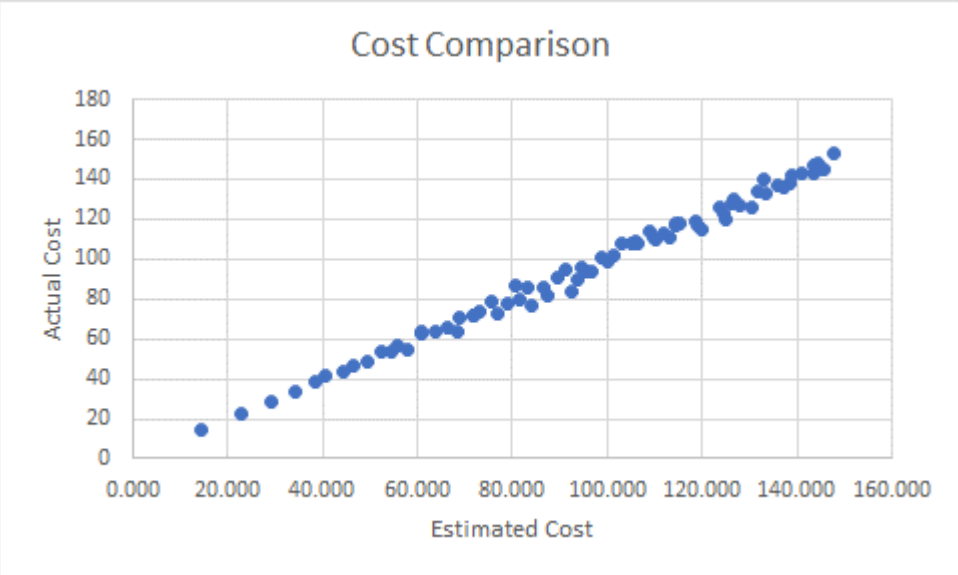
$$I^* = 883.25 \cdot Size^{0.35} \cdot \sigma^{0.76}$$

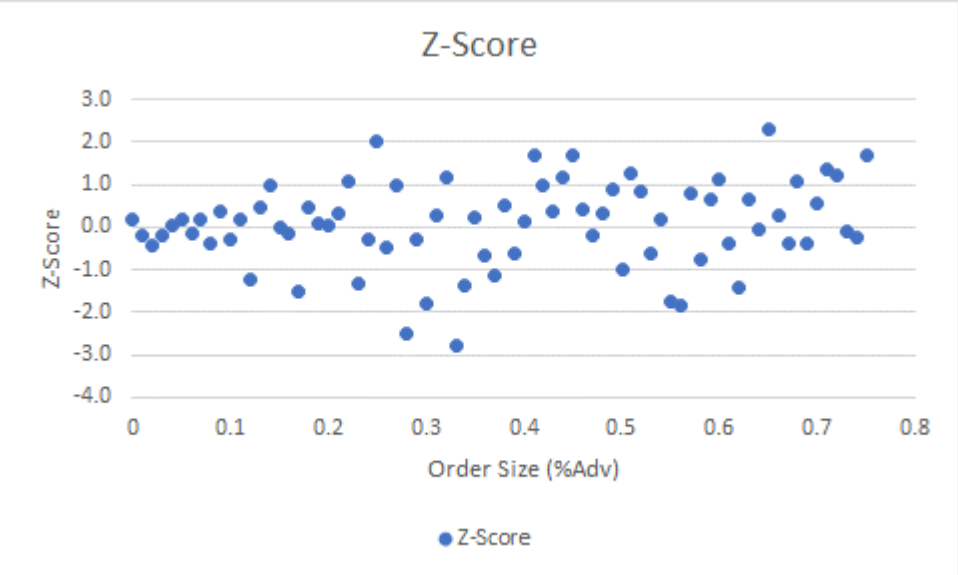
$$MI = I^* \cdot \left(0.96 \cdot POV^{0.83} + (1 - 0.96)\right)$$

Comparison of Actual to Estimated

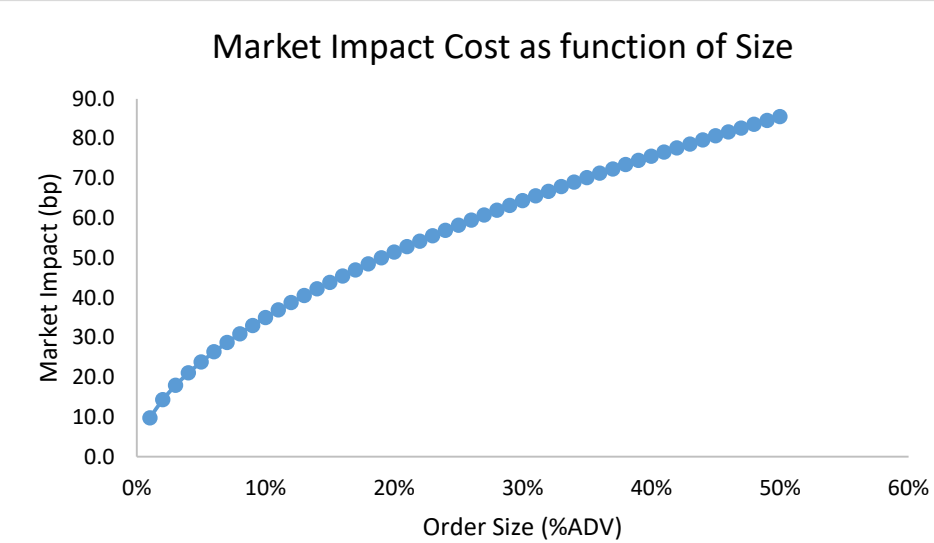


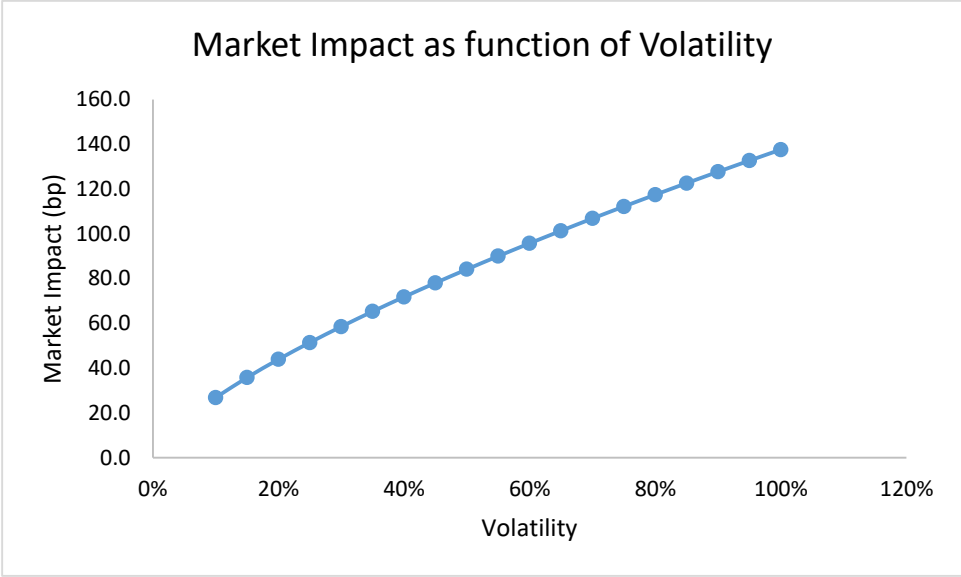


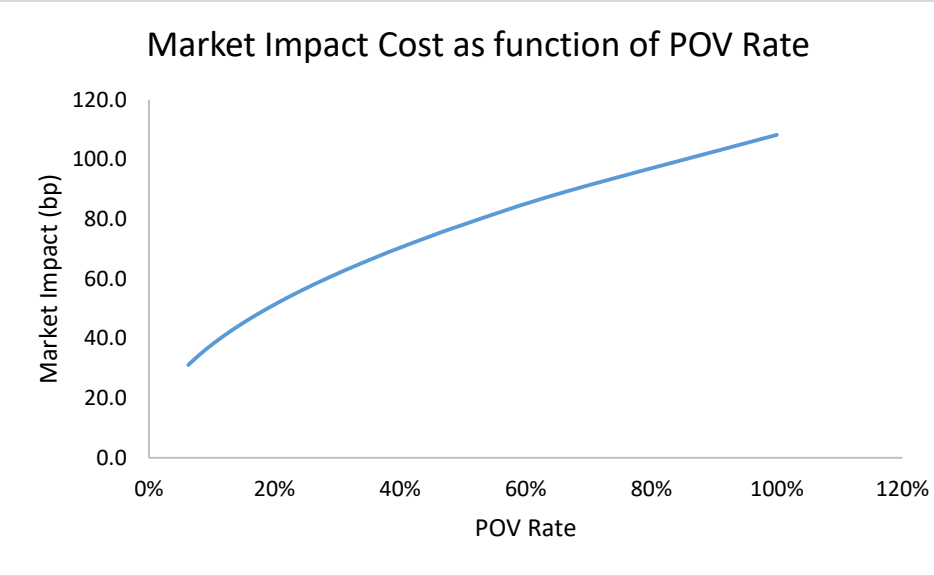


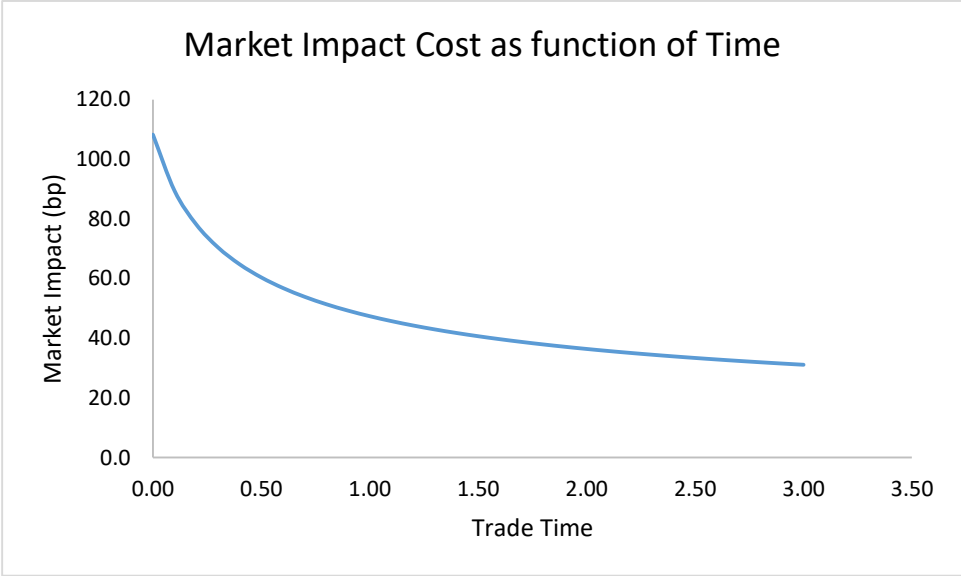


Market Impact Relationships







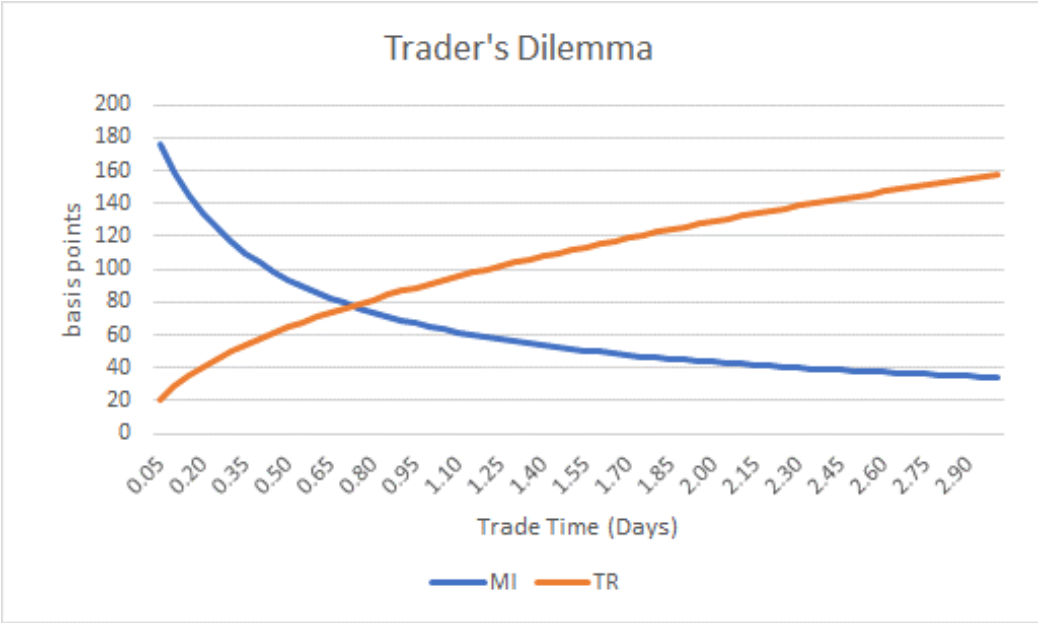


Traders Dilemma

Traders Dilemma

Traders Dilemma:

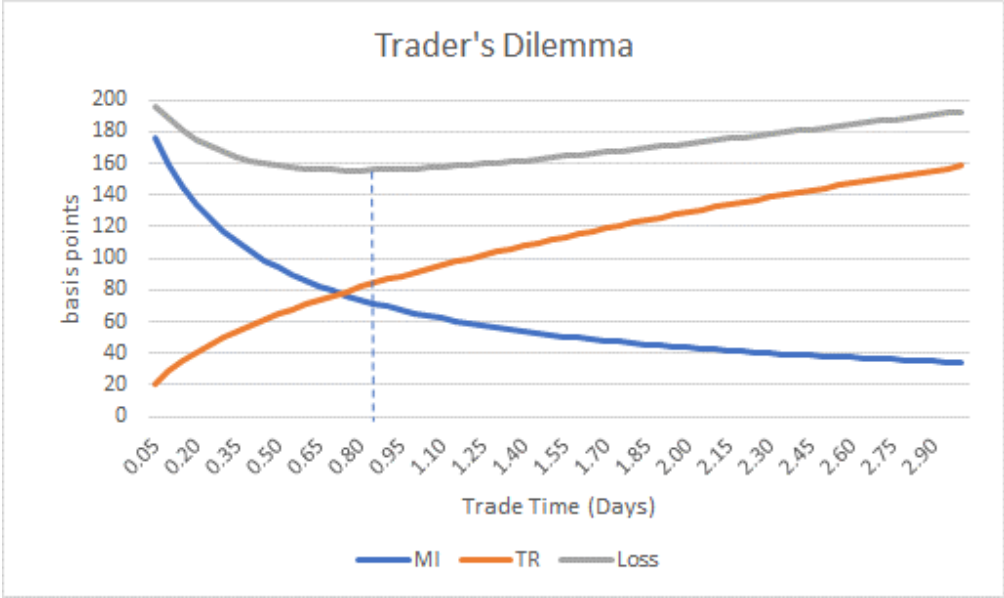
- *Trade too fast and I will move the market. Trade too slow and the market will move me.*
- *Market Impact and Timing Risk are conflicting expressions.*



Solving Trader Dilemma

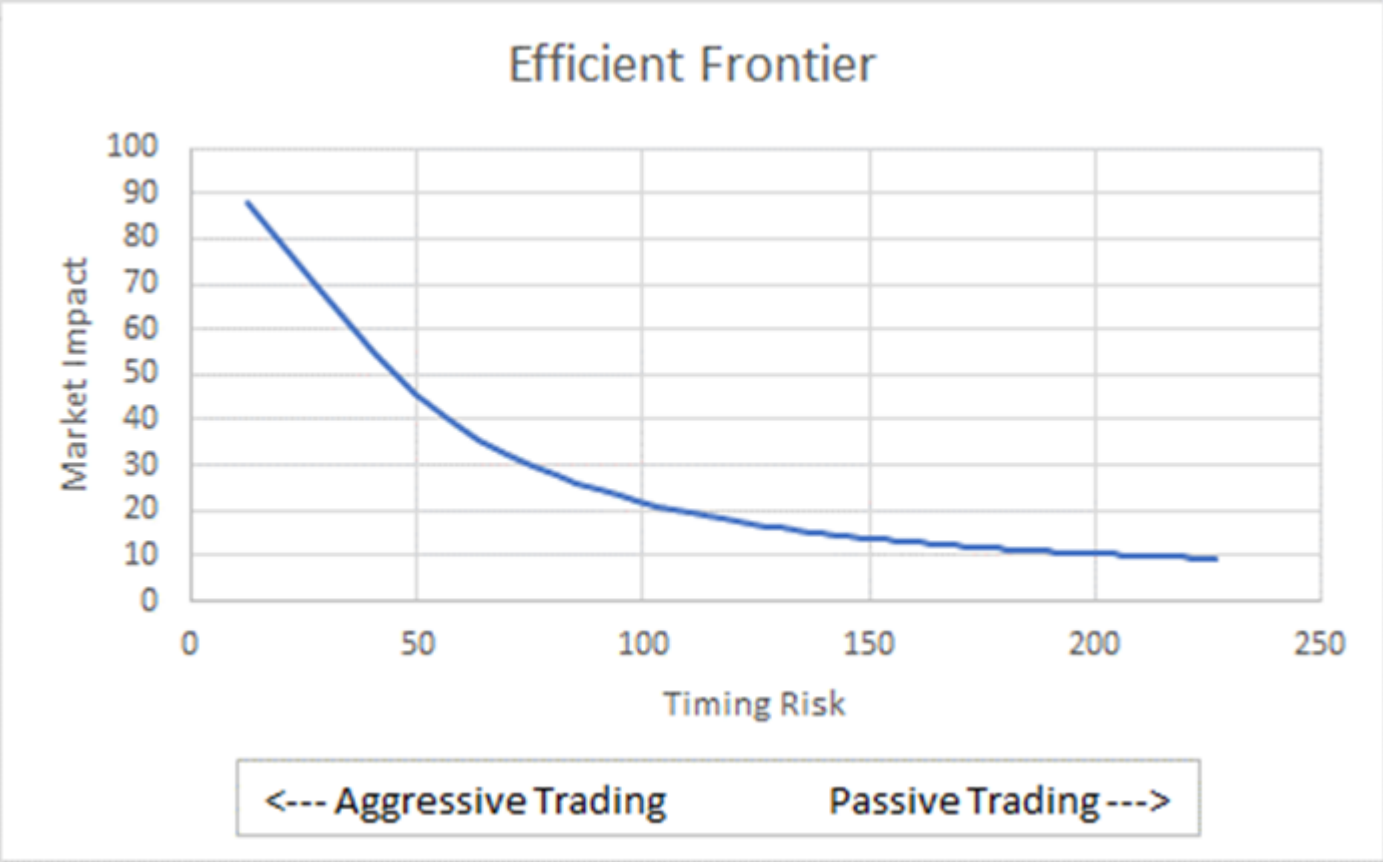
Solving Traders Dilemma:

- *Traders seek to manage the tradeoff between market impact cost (MI) and timing risk (TR) at a specified level of risk aversion λ.*
- *Lambda λ is specified by the investor and is often $0 \leq \lambda \leq 3$.*
- *Solve via Optimization*
  - *Solution provides optimal POV rate*
- *Each value of Lambda will have a different "Optimal" solution.*



$$Min \qquad MI + \lambda \cdot TR$$

Efficient Trading Frontier



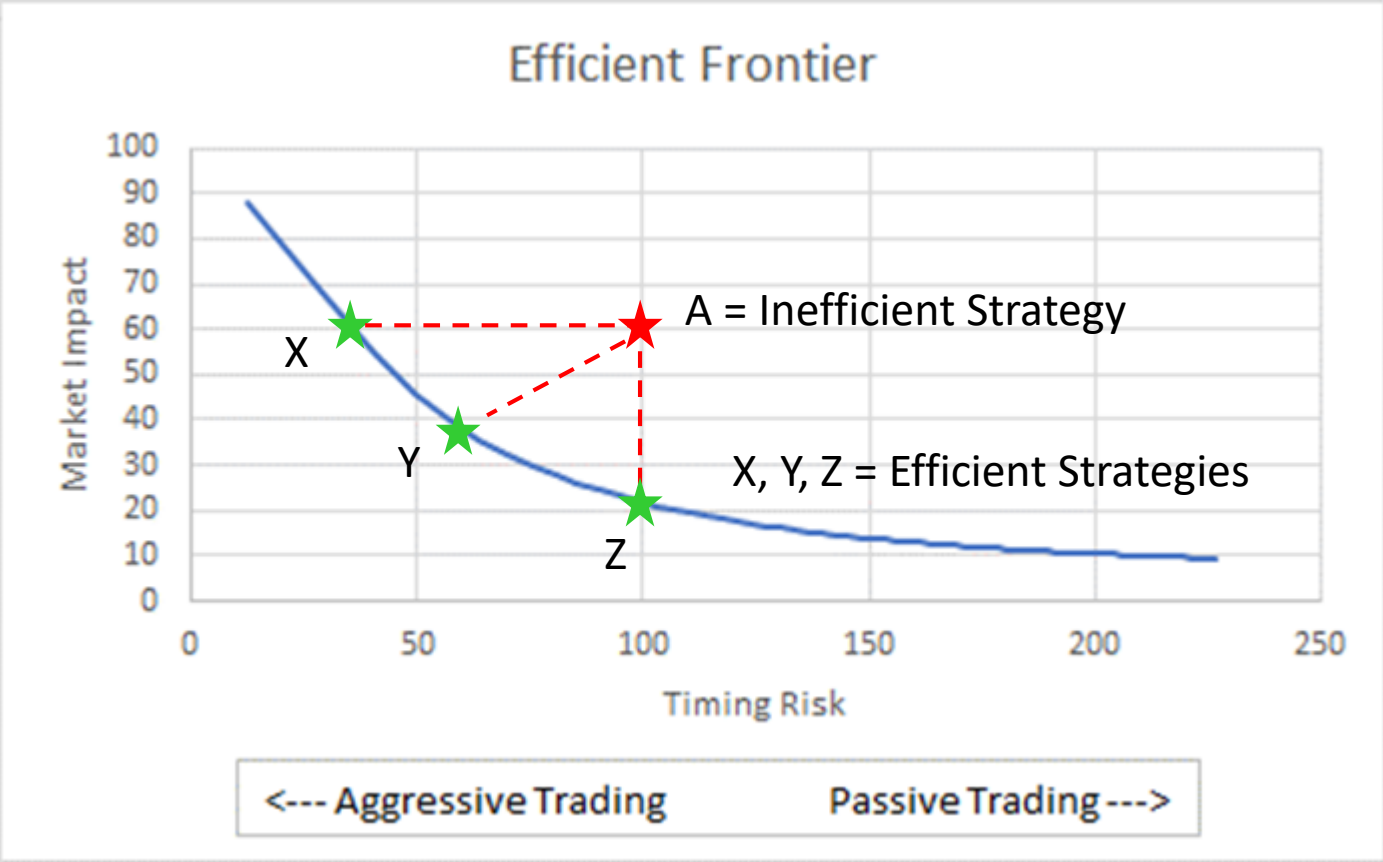
Optimization Results

Lambda	POV	Time	MI	TR
0.10	7%	1.25	14.6	142.9
0.25	14%	0.60	21.9	99.0
0.35	18%	0.45	25.9	85.7
0.50	25%	0.30	32.4	70.0
0.75	33%	0.20	40.1	57.2
1.00	50%	0.10	54.6	40.4
2.00	90%	0.01	87.7	12.8

Efficient Trading Frontier: Almgren & Chriss (1997)

Source: Kissell Research Group (2022)

Efficient Trading Frontier

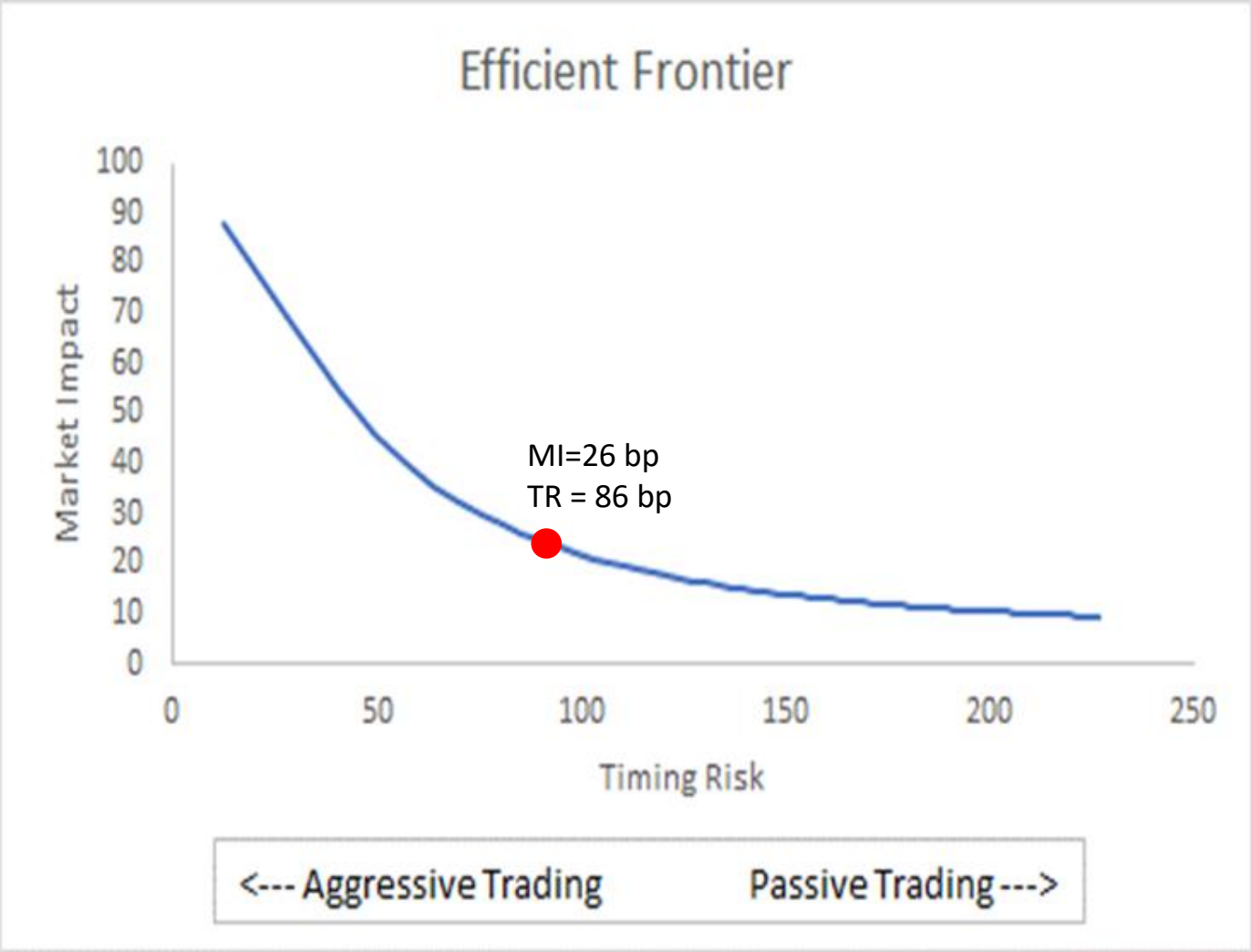


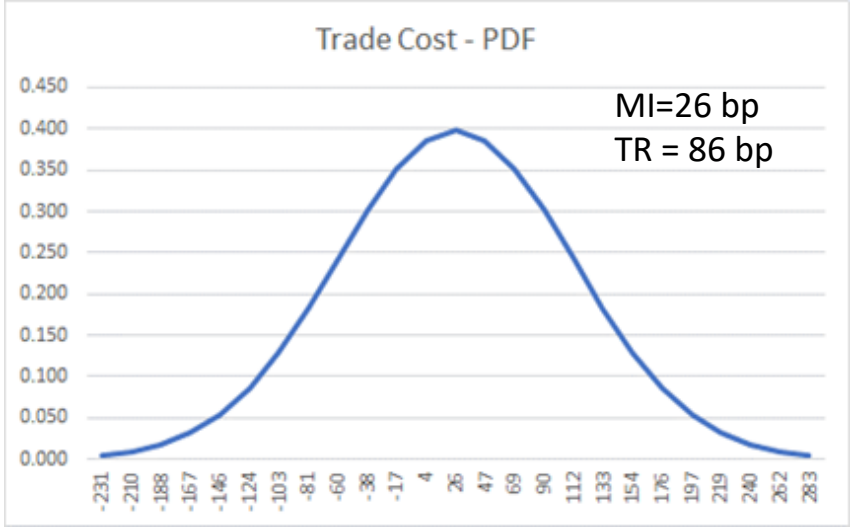
Optimization Results

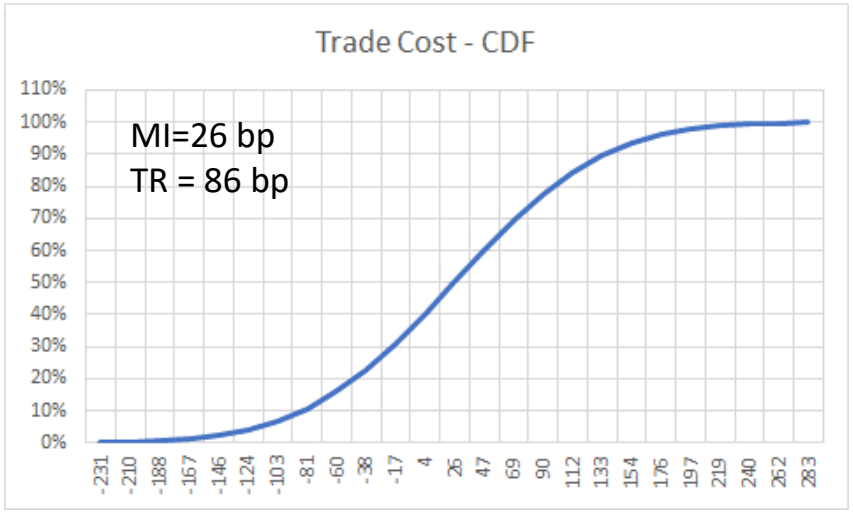
Lambda	POV	Time	MI	TR
0.10	7%	1.25	14.6	142.9
0.25	14%	0.60	21.9	99.0
0.35	18%	0.45	25.9	85.7
0.50	25%	0.30	32.4	70.0
0.75	33%	0.20	40.1	57.2
1.00	50%	0.10	54.6	40.4
2.00	90%	0.01	87.7	12.8

Source: Kissell Research Group (2022)

Where are the Expected Costs?







Source: Kissell Research Group (2022)

Example: Python

- Solve Traders Dilemma
- Optimization using Python
- Python Script:
- Using Estimated Market Impact Parameters

Example: Python #2

Python Trade Cost Scripts:

- KRG_Calc_TradeCosts.ipynb
- KRG_TraderDilemma.ipynb
- KRG_Optimal_POV.ipynb

Examples

Trade Cost Functions with MI Parameters

Parameter	Value
a1	883.25
a2	0.35
a3	0.76
a4	0.83
b1	0.96

$$\mathrm{I}^*(bp) = 883.35 \cdot Size^{0.35} \cdot \sigma^{0.76}$$

$$MI(bp) = 0.96 \cdot I^* \cdot POV^{0.83} + 0.04 \cdot I^*$$

$$TR(bp) = \sigma \cdot \sqrt{\frac{1}{250} \cdot \frac{1}{3} \cdot Size \cdot \frac{1 - POV}{POV}} \cdot 10^4_{bp}$$

- Trading Costs will be expressed in basis points (bp) unless otherwise noted.
- Market Impact and Timing Risk is always an expected value (e.g., forward looking).
- Market Impact Parameters are estimated via market data and non-linear regression techniques.
- Timing Risk is calculated based on Order Characteristics (Size and Volatility) and specified Trading Strategy (POV).
- Trading Strategy can be specified as: POV, Time, Schedule.

Questions

Q1) A trader received an order with the following characteristics: $Size = 15\%$, $\sigma = 25\%$, and $POV = 20\%$. What is the MI & TR of this order?

Q2) A trader received a buy order for 75,000 shares of stock RLK with instructions to transact the order over one-full day, e.g., 1-day VWAP Strategy. The statistics for RLK are: $ADV = 500{,}000$ and $\sigma = 25\%$. What is the MI & TR of this order?

Q3) A trader received an order to sell 100,000 shares of RLK using a POV=15% strategy. The stock characteristics are: $ADV = 1{,}000{,}000$ and $\sigma = 20\%$.

- How long will it take to trade this order (e.g., Time=Volume Time)?
- What is the MI & TR of this order?

Q4) A trader received an order with the following characteristics: $Size = 15\%$, and $\sigma = 25\%$. The trader has a risk aversion of $\lambda = 0.35$. What is the optimal POV rate that balances the tradeoff of MI & TR of this order at the specified level of risk aversion?

Q5) A trader will execute the order using a POV=20% strategy during a period where the expected period volume is 50,000 shares. How The stock has ADV=1,000,000 shares. How many shares will the trader during this period?

Parameter	Value
a1	883.25
a2	0.35
a3	0.76
a4	0.83
b1	0.96

Trade Cost Equations:

$$\mathrm{I}^*(bp) = a_1 \cdot Size^{a2} \cdot \sigma^{a3}$$

$$MI(bp) = b_1 \cdot I^* \cdot POV^{a4} + (1 - b_1) \cdot I^*$$

$$TR(bp) = \sigma \cdot \sqrt{\frac{1}{250} \cdot \frac{1}{3} \cdot Size \cdot \frac{1 - POV}{POV}} \cdot 10^4_{bp}$$

$$Time = Size \cdot \frac{1 - POV}{POV}$$

Trading Analytics

Trading Analytics

Pre-Trade

- Cost Curves
- Basket Analysis

Intra-Day

- Real-Time

Post-Trade

- Cost Curves
- Basket Analysis

Demonstrate how to develop each analytic in Excel using KRG Analytics Suite

Pre-Trade Analysis

Pre-Trade Analysis: Pre-trade analysis provides investors with the estimated trading costs of an order. This includes the expected market impact, price appreciation, and timing risk. Traders also use pre-trade analysis to evaluate different trading strategies and algorithms based on trading cost and risk. They also allow investors to incorporate their own market views and proprietary alpha forecasts directly into the analyses so that investors can develop a customized analysis for their specified order. Traders use these functions to perform single stock and portfolio multi-period trade schedule optimization.

- The goal of pre-trade TCA is to provide traders with the necessary information to select the most appropriate algorithm and strategy based on the underlying investment objectives of the fund.

Pre-Trade Analysis

Pre-Trade Model

MI Parameters	
a1=	746.1864
a2=	0.3521
a3=	0.7776
a4=	0.9100
b1=	0.9500

Strategy	POV	Time	MI	TR
POV	10%	2.1	17.0	80.6
Time	16%	1.0	27.2	61.8
Optimal	11%	1.6	18.8	73.4

						POV Strategy				VWAP Strategy				Single Stock Optimization			
Num	Symbol	Side	Shares	Weight	Size	POV	Time	MI	TR	POV	Time	MI	TR	POV	Time	MI	TR
1	A	Buy	415,479	3.46%	15.00%	10%	1.35	17.3	79.0	13.0%	1.0	20.60	79.04	11.5%	1.15	18.93	73.09
2	AAL	Sell	111,250	0.11%	0.47%	10%	0.04	6.8	20.3	0.5%	1.0	2.34	20.30	8.5%	0.05	6.16	22.20
3	AAP	Buy	125,666	1.59%	1.06%	10%	0.10	6.7	20.8	1.0%	1.0	2.63	20.77	8.5%	0.11	6.09	22.72
4	AAPL	Buy	47,676	0.46%	4.07%	10%	0.37	13.2	52.7	3.9%	1.0	7.91	52.69	10.5%	0.35	13.65	51.28
5	ABBV	Buy	497,060	3.59%	1.06%	10%	0.10	5.8	17.1	1.1%	1.0	2.25	17.05	8.5%	0.11	5.22	18.65
6	ABC	Buy	293,879	2.35%	3.56%	10%	0.32	10.0	36.4	3.4%	1.0	5.64	36.44	9.5%	0.34	9.66	37.49
7	ABMD	Buy	36,839	0.84%	1.85%	10%	0.17	7.5	24.3	1.8%	1.0	3.35	24.33	9.0%	0.19	6.99	25.79
8	ABT	Buy	430,812	3.87%	8.10%	10%	0.73	13.5	55.9	7.5%	1.0	11.33	55.94	10.5%	0.69	13.94	54.44
9	ACN	Buy	182,047	3.40%	5.68%	10%	0.51	13.3	54.0	5.4%	1.0	9.29	53.96	10.5%	0.48	13.74	52.52
10	ADBE	Buy	7,914	0.32%	0.32%	10%	0.03	5.4	14.8	0.3%	1.0	1.78	14.79	8.0%	0.04	4.71	16.71
11	ADI	Buy	6,362	0.06%	0.19%	10%	0.02	4.6	11.7	0.2%	1.0	1.47	11.75	7.5%	0.02	3.86	13.75
12	ADM	Buy	1,072,359	4.12%	45.30%	10%	4.08	25.2	135.2	31.2%	1.0	57.20	135.16	13.0%	3.03	29.95	116.55
13	ADP	Buy	132,582	1.53%	3.69%	10%	0.33	12.4	48.3	3.6%	1.0	7.11	48.26	10.0%	0.33	12.40	48.26
14	ADSK	Sell	6,105	0.12%	0.32%	10%	0.03	5.7	15.7	0.3%	1.0	1.87	15.67	8.0%	0.04	4.93	17.72
15	AEE	Buy	571,123	3.73%	113.17%	10%	10.18	31.0	184.0	53.1%	1.0	108.33	183.99	14.0%	6.95	38.73	152.00
16	AEP	Buy	812,402	5.48%	33.48%	10%	3.01	16.5	77.3	25.1%	1.0	31.63	77.31	11.5%	2.58	18.07	71.49
17	AES	Buy	404,373	0.60%	17.02%	10%	1.53	20.6	99.6	14.5%	1.0	26.45	99.58	12.0%	1.25	23.19	89.89
18	AFL	Buy	216,708	0.65%	16.32%	10%	1.47	20.7	100.2	14.0%	1.0	25.97	100.24	12.0%	1.20	23.35	90.48
19	AIG	Sell	323,408	0.74%	12.13%	10%	1.09	22.5	109.6	10.8%	1.0	23.64	109.63	12.0%	0.89	25.31	98.96
20	AIV	Buy	329,337	0.92%	6.26%	10%	0.56	13.9	57.3	5.9%	1.0	10.18	57.32	10.5%	0.53	14.35	55.79

Intra-Day Analysis

Intra-Day Analysis: Intra-day analysis provides investors with the necessary real-time analytics to monitor transaction costs during trading. These models provide investors with point in time trading costs estimates (for executed shares) and the projected trading costs that will result from completing the order (for those shares that still need to be executed). This is accomplished by incorporating market momentum and actual market conditions (volume, volatility, and aggregated imbalances) directly into the analysis.

- The goal of Intra-Day analysis is to provide investors with the necessary information to determine when it is advantageous to take advantage of changing market conditions and favorable opportunities.

Cost Curve

Cost Curve: A cost curve provides the Portfolio Manager with insight into the cost of executing a specified number of shares utilizing a specified trading strategy commonly expressed in terms of percentage of volume (POV) or trade time (Time).

- The goal of cost curves is to provide investors with the necessary information to determine how many shares can be absorbed into the market at the managers level of price sensitivity.

Cost Curves

Cost Curves

Order	
Symbol:	A
Volatility:	0.186
ADV:	2,770,507
Price:	100.94

Order Statistics			Market Impact Estimates - basis points									
Size			POV Strategy									
(% Adv)	Shares	Dollars	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
1%	27,705	2,796,550	4.5	6.7	8.7	10.8	12.7	14.7	16.6	18.5	20.3	22.2
5%	138,525	13,982,749	7.9	11.7	15.4	19.0	22.5	25.9	29.2	32.6	35.8	39.1
10%	277,051	27,965,498	10.1	15.0	19.7	24.2	28.7	33.0	37.3	41.6	45.7	49.9
15%	415,576	41,948,246	11.6	17.3	22.7	27.9	33.1	38.1	43.0	47.9	52.8	57.6
20%	554,101	55,930,995	12.9	19.1	25.1	30.9	36.6	42.1	47.6	53.0	58.4	63.7
25%	692,627	69,913,744	13.9	20.7	27.2	33.4	39.6	45.6	51.5	57.4	63.2	68.9
30%	831,152	83,896,493	14.8	22.1	29.0	35.7	42.2	48.6	54.9	61.2	67.4	73.5
35%	969,677	97,879,242	15.7	23.3	30.6	37.6	44.5	51.3	58.0	64.6	71.1	77.6
40%	1,108,203	111,861,991	16.4	24.4	32.0	39.5	46.7	53.8	60.8	67.7	74.5	81.3
45%	1,246,728	125,844,739	17.1	25.5	33.4	41.1	48.7	56.1	63.4	70.6	77.7	84.7
50%	1,385,254	139,827,488	17.8	26.4	34.7	42.7	50.5	58.2	65.8	73.2	80.6	87.9
55%	1,523,779	153,810,237	18.4	27.3	35.9	44.1	52.2	60.2	68.0	75.7	83.4	90.9
60%	1,662,304	167,792,986	18.9	28.2	37.0	45.5	53.8	62.0	70.1	78.1	86.0	93.8
65%	1,800,830	181,775,735	19.5	29.0	38.0	46.8	55.4	63.8	72.1	80.3	88.4	96.4
70%	1,939,355	195,758,484	20.0	29.7	39.0	48.0	56.9	65.5	74.0	82.4	90.8	99.0
75%	2,077,880	209,741,232	20.5	30.5	40.0	49.2	58.3	67.1	75.8	84.5	93.0	101.4
80%	2,216,406	223,723,981	21.0	31.2	40.9	50.4	59.6	68.7	77.6	86.4	95.1	103.8
85%	2,354,931	237,706,730	21.4	31.8	41.8	51.4	60.9	70.1	79.3	88.3	97.2	106.0
90%	2,493,456	251,689,479	21.8	32.5	42.6	52.5	62.1	71.6	80.9	90.1	99.2	108.2
95%	2,631,982	265,672,228	22.3	33.1	43.5	53.5	63.3	72.9	82.4	91.8	101.1	110.2
100%	2,770,507	279,654,977	22.7	33.7	44.3	54.5	64.5	74.3	83.9	93.5	102.9	112.2

Post-Trade Analysis

Post-Trade Analysis: Post-trade analysis serves as a report card on the trade. It provides investors with the cost of the trade and an evaluation of the trading performance. Post-trade analysis will include a cost comparison to various price benchmarks (e.g., Arrival, Open, VWAP, Close, T-1, and T+1) as will also compare actual costs to a pre-trade trading cost estimate for the selected strategy.

Post-trade analysis allows funds to determine how well their brokers and algorithms performed given actual market conditions. Investors can rank their brokers and their algorithms to determine which brokers and algorithms are adding value to the fund and determine which algorithms and which brokers may be underperforming expectations and causing the fund to incur unnecessary higher trading costs. Investors can compute additional statistics related to the trade such as the relative performance measure (RPM) to help determine which brokers are adding value to the trading process and which brokers are causing funds to incur unnecessary trading costs. Customized post trade reports provide clients with the ability to sort, filter, and evaluate different trading situations right on their own desktop.

- The goal of Post-Trade analysis is to help investors measure trading costs and evaluate trade performance.
- It also helps investors determine which broker and which broker algorithms provide the best results based on order characteristics and market conditions.

Post-Trade Analysis

Post-Trade Analysis (TCA)

	TCA: Benchmark Cost Analysis						TCA: Implementation Shortfall					
Broker	Open Cost (bp)	Arrival Cost (bp)	VWAP Cost (bp)	Close Cost (bp)	Value-Add (bp)	Z-Score	Delay Cost ($)	Exec. Cost ($)	Opportunity Cost ($)	Commission Cost ($)	Total IS ($)	Total IS (bp)
Broker A	14.3	14.3	42.0	-20.3	4.4	0.09	0	-69,238	21,157	16,817	-31,264	-1.39
Broker B	-30.2	-30.2	25.1	37.9	-4.7	-1.54	0	-135,751	0	16,303	-119,449	-14.98
Broker C	-10.0	-10.0	28.4	18.4	8.3	0.42	0	-309,788	164,834	41,577	-103,377	-2.94
Broker D	-0.3	-0.3	-9.7	-31.4	-7.0	0.13	0	140,425	24,057	22,181	186,662	6.43
Broker E	-18.2	-18.2	12.3	73.0	11.5	2.83	0	-493,213	398	28,164	-464,651	-17.53

Order Details				TCA: Benchmark Cost Analysis						TCA: Implementation Shortfall					
Num	Symbol	Side	Broker	Open Cost (bp)	Arrival Cost (bp)	VWAP Cost (bp)	Close Cost (bp)	Value-Add (bp)	Z-Score	Delay Cost ($)	Exec. Cost ($)	Opportunity Cost ($)	Commission Cost ($)	Total IS ($)	Total IS (bp)
1	A	Buy	Broker C	-23.8	-23.8	92.2	78.1	41.1	0.52	0	-99,715	0	4,155	-95,560	-22.8
2	AAL	Sell	Broker D	-8.1	-8.1	48.5	-224.4	14.4	0.80	0	890	-5,785	890	-4,005	-29.3
3	AAP	Buy	Broker C	-28.0	-28.0	42.6	386.8	34.8	1.67	0	-54,036	0	1,257	-52,780	-27.4
4	AAPL	Buy	Broker B	-1.7	-1.7	26.8	-213.0	15.0	0.28	0	-954	0	477	-477	-0.9
5	ABBV	Buy	Broker E	-59.4	-59.4	-20.6	61.2	65.1	3.82	0	-258,471	0	4,971	-253,500	-58.2
6	ABC	Buy	Broker E	-29.9	-29.9	12.4	150.2	39.9	1.10	0	-85,225	0	2,939	-82,286	-28.9
7	ABMD	Buy	Broker E	52.3	52.3	68.7	42.5	-45.0	-1.89	0	51,280	398	354	52,032	51.0
8	ABT	Buy	Broker A	6.4	6.4	48.9	149.1	6.9	0.13	0	29,252	-19,774	4,179	13,657	2.9
9	ACN	Buy	Broker C	-36.7	-36.7	-106.3	96.4	50.0	0.93	0	-151,099	0	1,820	-149,279	-36.3
10	ADBE	Buy	Broker A	-5.3	-5.3	-36.4	111.8	10.7	0.73	0	-2,037	-450	78	-2,408	-6.2
11	ADI	Buy	Broker A	-16.3	-16.3	-75.8	-112.0	20.9	1.78	0	-1,209	0	64	-1,145	-15.4
12	ADM	Buy	Broker C	-38.7	-38.7	-89.9	355.5	63.9	0.47	0	-193,025	0	10,724	-182,301	-36.6
13	ADP	Buy	Broker D	39.4	39.4	117.0	-58.2	-27.0	-0.56	0	72,920	0	1,326	74,246	40.1
14	ADSK	Sell	Broker D	0.9	0.9	44.8	289.2	4.8	0.31	0	-122	0	61	-61	-0.4
15	AEE	Buy	Broker D	-19.0	-19.0	-157.1	232.0	49.9	0.27	0	-85,668	0	5,711	-79,957	-17.7